NUMERICAL METHODS AND COMPLEX VARIABLES

QUESTION BANK

UNIT-I:NUMERICAL METHODS

1.By the fixed point iteration process, find the root correct to 3-decial places, of the equation x=cosx, near x=π/4.

2.By the single point iteration method, find the root of the equation x^3-2x—5=0 which lies near x=2.

3.Find the positive root of x^4-x-10=0 by iteration.

4.Find the value of y for x=0.4 by Picard's method, given that $\frac{dy}{dx} = x^2 + y^2$,y(0)=0.

5.Solve $\frac{dy}{dx} = 2x - y$,y(1)=3 by Picard's method.

6.Evaluate the values of y(1.1) and y(1.2) from $y'' + y^2y' = x^3$,y(1)=1,y′(1)=1 by taylor series method.

7.Use Runge-Kutta method to find y(0.1) for the equation $y'' + xy' + y =$ 0,y(0)=1,y'(0)=0.

8.Find the first and second derivatives of the function tabulated below at the point x=1.5

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.0	13.625	24.0	38.875	59.0

UNIT-II: **LAPLACE TRANSFORMS**

1.Find $L\left\{\frac{\sin 3t \cos t}{t}\right\}$, Using Laplace transform,

2.Evaluate $\int_0^\infty t^2 e^{-4t} \sin 2t \, dt$ Using Laplace transform

3.Using the convolution theorem find $L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$

4.Using Laplace transform, solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$, given that y(0)=0, $y^1(0)=1$.

5.Solve the following differential equation using the Laplace transform $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=5\, sint\ given\ that\ y(0)=0, y'(0)=0$.

6.Define an inverse Laplace transform of a function

7. If $L\{f(t)\}=\frac{9s^2-12s+15}{(s-1)^3}$,find $L\{f(3t)\}$ using change of scale property

8.. Using the convolution theorem find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

9. Using Laplace transform, solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$, given that y(0)=0, $y^1(0)=1$.

UNIT-III: **ANALYTICAL FUNCTIONS**

1.Show That the function is defined by $f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ at $z\neq 0$, and f(0) = 0 is continuous and satisfies C-R equations at the origin but $f'(0)$ does not exist.

2.Find the analytic function whose real part is $e^{2x}(xCos2y-ySin2y)$.

3. Find analytical function whose real part is $r^2Cos2\theta+rSin2\theta$.

4.If f (z) is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f^1(z)|^2$.

5.State and Prove Cauchy's Integral Formula.

6.Evaluate $\int_C \frac{z^2-z+1}{z-1}dz,\ wherec: |z|=\frac{1}{2}$.

7.Evaluate $\int_C \frac{logz}{(z-1)^3}dz, wherec: |z-1|=\frac{1}{2}$ using Cauchy's Integral Formula.

8.Evaluate $\int_C \frac{z+4}{z^2+2z+5}dz, wherec: |z+1-i|=2$.

UNIT-IV: **SINGULARITIES AND RESIDUES**

1.Define (i) Removable singularity, (ii) Essential singularity, (iii) Pole Singularity.

2.Find the Laurent's Series of $f(z)=\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region$3<|z+2|<5$.

3.Evaluate by Residue Theorem $\int_C \frac{z-1}{(z+1)^2(z-2)}dz, wherec: |z-i|=2$.

4.Evaluate $\int_0^{2\pi}\frac{d\theta}{5-3Cos\theta}$ by Contour Integration.

5.Find the Laurent's Series of $\frac{1}{z^2-4z+3}$ for $1<|z|<3$.

6.Find the Taylor's Series of e^z about z = 3.

7.Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where c is the Circle given by

(i).$|z| = 1$, (ii). $|z + 1 - i| = 2$, (iii). $|z + 1 + i| = 2$

8.Expand $\frac{7z-2}{(z+1)z(z-2)}$ about the point z = -1 in the region $1 < |z + 1| < 3$ as Laurent's Series.

9.Expand f(z) = Cosz in Taylor's Series about $z = \frac{\pi}{4}$.

10.State and Prove Cauchy's Residue Theorem.

UNIT-V: **CONFORMAL MAPPING**

1.Find and plot the image of the regions (i) $x > 1$ (ii) $y > 0$ (iii) $0 < y < \frac{1}{2}$ [14M]

Under the transformation $w = \frac{1}{z}$.

2.Find the Fixed Points of the Transformation.

(i). $w = \frac{2i-6z}{iz-3}$ (ii). $w = \frac{6z-9}{z}$ (iii). $w = \frac{z-1}{z+1}$ (iv). $w = \frac{2z-5}{z+4}$.

3.Define Bilinear Transformation and Show That Every Bilinear Transformation

is Conformal.

4.Find the Bilinear Transformation which maps the points (0, 1, i) into the points (1+i,-i,2-i).

5.Write Cross-Ratio of four points z_1, z_2, z_3, z_4.

6.Show that the function w = $\frac{4}{z}$ transforms the straight line x = c in the z – plane into a circle in the w-plane.

7.Define Critical Point and Bilinear Transformation.

8.Show that the function $w = \frac{4}{z}$ Transforms the line x = c in the z- plane into a Circle in the w- plane.

9.Under the Transformation $w = \frac{z-i}{1-iz}$ find the image of the Circle

(i). $|w| = 1$, (ii). $|z| = 1$.

10..Find the Bilinear Transformation which maps 1 + i, - i, 2 - i of the z- plane into the points0, 1, i respectively of the w-plane. Find the Fixed and Critical Points of this Transformation.